which splits a shock to higher pressure into two. In fact, if the material undergoing a phase change is also elastic-plastic, a three wave structure may be produced, as in iron, illustrated in Fig. 18.

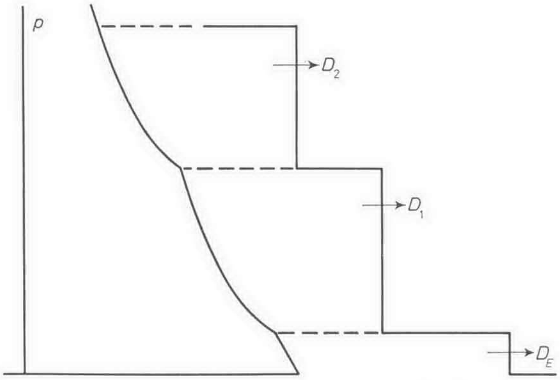

Fig. 18. – Three-wave structure in iron.

The phase transition is also responsible for a new phenomenon, the rarefaction shock. The unloading curve for an elastic-plastic material is always concave upward, the condition required for a rarefaction to spread as it progresses. But the unloading curve for an equilibrium phase transition of the kind shown in Fig. 17 has a convex upward region and this is responsible for producing a rarefaction shock. This is illustrated in Fig. 19.

The unloading curve is shown in Fig. 19 *a*) with two cusps, one at B and one at D. Suppose the material has been uniformly shocked to point $C(u_2, p_2)$,

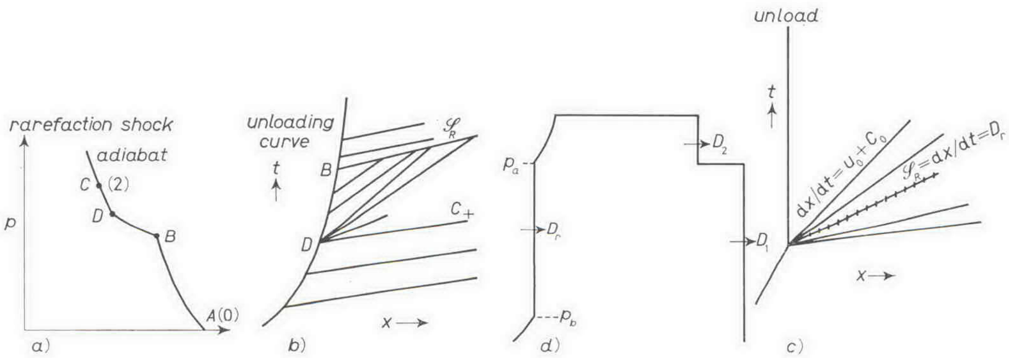

Fig. 19. – Rarefaction shock.

and that the pressure on the driving surface is slowly reduced so that the trace of the surface in the (x, t) plane is the curve BD shown in Fig. 19 *b*). At each decrement in driving pressure the free surface can be thought to send out a disturbance traveling along a $C+$ characteristic with velocity $u+c$. When the unloading process has progressed until the surface pressure has decreased from C to D, a singularity in the sound velocity c is encountered. At D we suppose c to take on all values from the second phase value on CD at D to the mixed phase value on DB at D. Then the point D on the unloading curve in Fig. 19 *b*) is the source of a fan of $C+$ characteristics as shown. The slower sound velocities correspond to the steeper lines. From D to B the sound velocities are all small and the $C+$ characteristics are steep. At B a second singularity in c is encountered, but now the characteristics, instead of forming a fan as at D, crowd into one another, intersecting one another, and ultimately intersecting some of the characteristics from D. An intersection of two characteristics means that the field variables carried along the characteristics are multiple-valued at that point. In Fig. 19 *b*) the build-up of shock amplitude occurs over some distance as more and more characteristics intersect. In Fig. 19 *c*) we suppose that the driving pressure is released suddenly and the fully developed shock, $\mathscr{S}_R$, radiates from the corner in the unloading path as shown. In Fig. 19 *d*) is shown schematically the structure of a wave including a double compression shock and a rarefaction shock. The pressures p_a and p_b are determined by the condition that

$$(u+c)_a = D_R = (u+c)_b \,, \tag{65}$$

where D_R is the velocity of the rarefaction shock.

Some mechanical effects of the rarefaction shock will be described in Sect. 8.

6. – Stress-relaxation in elastic-plastic solids.

In order to provide a suitable framework for discussing the constitutive relations for a stress-relaxing solid, it is necessary to introduce some general concepts from continuum mechanics. We consider again an element of mass in a flow field, subject to forces of acceleration and compression transmitted through its immediate neighbors. We are interested in the response of this mass element to the stresses transmitted across its boundaries, and in order to discuss them we choose a co-ordinate system (x_1, x_2, x_3) which diagonalizes the stress and strain matrices. The principal stresses and strains are $\{\sigma_i\}$ and $\{\varepsilon_i\}$ respectively, with $i=1, 2, 3$. We define a set of «stress deviators», S_i, and «strain deviators», E_i:

$$S_i = \sigma_i + \overline{p} \,, \tag{66}$$

$$E_i = \varepsilon_i - \theta/3 \,, \tag{67}$$

where $\overline{p} = -(\sigma_1+\sigma_2+\sigma_3)/3$, $\theta = \varepsilon_1+\varepsilon_2+\varepsilon_3$.
The S_i incorporate all the stress of deformation; the E_i are the strains of deformation. For purely elastic strain we can write Hooke's law in incre-